# Quantization Error Analysis of Fractal Image Coding Based on State-Space Approach(状態空間法に基づくフラクタル画像符号化の量子化誤差解析)

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# 論　文　内　容　要　旨

## Chapter 1. Introduction

This chapter addresses the motivation and the purposes of this research work. Outline of this dissertation is also given.

Due to the redundancy of image data and the human perception of image data, it has been reported that various lossless/lossy image coding schemes are possible. Among the image coding schemes, fractal image coding has seen much attention as a novel coding scheme due to the multiresolution feature and high compression rate comparable to the previous coding methods such as wavelet coding or JPEG and so on.

In practical sense, the quantization error generated during decoding due to the finite wordlength is very important when implementing a decoder with finite wordlength. To the author's knowledge, however, the quantization error during decoding in fractal image coding has not been reported.

This dissertation presents analysis methods of quantization error of fractal image coding based on state-space approach. There are two types of quantization error in fractal image coding. One is roundoff error and the other is coefficient-quantization error. This thesis uses the state-space approach. The state-space approach is an established method in digital signal processing areas such as digital filter theory and control theory.

## Chapter 2. Fundamental study of fractal image coding and state-space approach

This chapter reviews some fundamental theories of fractal image coding and state-space approach in order to make this thesis to be self-contained. This chapter is divided into two parts. One part is about some conventional algorithms of encoding and decoding of fractal image coding. Partition and classification concepts in encoding process are also described. Various coding schemes according to the different partition schemes are just referred. The other part is some brief reviews about the state-space approach, which has been established in signal processing area such as in the digital filter theory or control theory. This analysis method is used in the analysis of roundoff error and coefficient quantization error of fractal image coding.

## Chapter 3 . Roundoff error analysis of fractal image coding

This chapter describes roundoff error analysis of fractal image coding. First, a state-space model of roundoff error is given with some modifications from conventional state-space model. This chapter also shows how the deterministic iterated function system for fractal image coding can be described in the state space. Next, the theory of analysis follows. Next, the experimentation results are given to validate the analysis theory. Last, a summary is given with some discussions on the obtained results and the limitation that this analysis method includes.

In this chapter, we derive the output error variance matrix as

$$V_\varepsilon = \sigma^2 G = \sigma^2 (m+1) W^t,$$

where $\sigma^2 = 2^{-2L}/12$, $W$ and $G$ are the error (noise) matrix and the error power gain, respectively. We note that the output error variance matrix, in our analysis, can be represented simply by three terms : $\sigma^2$ that depends on the finite wordlength (bits), $m+1$ that is 1 plus the decimation ratio $m$ and $W^t$ that depends on the scaling factors.

For the measurements of roundoff error, we also define the output error variance $\bar{V}_\varepsilon(k)$ at iteration $k$ as follows :

$$\bar{V}_\varepsilon(k) = \frac{\mathrm{tr}(V_\varepsilon)}{n^2} .$$

In order to verify the analysis, we have tested the method on a 32×32 down-sampled Lena image with 256 grey levels. Experimental results show that our modeling and analysis are valid. In practical meaning, however, the analysis method described in this chapter cannot be applied to the large images such as 256×256 images, becauseof the exhaustive computation of $n^2 \times n^2$ size matrices.

## Chapter 4 . Roundoff error analysis based on a simplified state-space model

This chapter presents a roundoff error analysis method based on a simplified state-space model. First, the simplified model is described for the roundoff error analysis. Some assumptions for the simplification are given with the detailed reasons. Two methods are tried for the coefficient-quantization error analysis. One is deterministic method and the other is statistical method. The statistical method is successful but the deterministic method has failed. The detailed description is given with corresponding modifications from the previous chapter. In addition, the comparisons of simulation results with the analysis results are made for various types of standard images in order to verify the analysis method experimentally. Last, a summary is given for the obtained results.

In this chapter, we use a simplified state-space model for the decoding. This model can be applied to large images such as 256×256 size images and 512×512 images, because the result of analysis is given by a simple equation which consists of only the scaling coefficients for an image. While the model in the previous chapter describes the decimation and scaling of grey levels by a scaling matrix $\boldsymbol{A}$, the model in this chapter describes the decimation and scaling of grey levels by $(1/m)$ $\boldsymbol{M}$ and $\boldsymbol{S}$ respectively. By ignoring the effect of $1/m$, a simple result can be obtained, which is experimentally verified by testing on the various type of standard images.

As an analysis result, this method describes the output error variance as a simple equation as follows :

$$\bar{V}_\varepsilon(k) = \frac{\sigma^2}{n^2} \sum_{l=1}^{n^2} \left( \frac{1-s_l^{2k}}{1-s_l^2} - 1 \right) ,$$

which consists of scaling coefficients $s_l$, iteration number $k$, and wordlength $L$ in $\sigma^2$. This equation does not need to be computed iteratively for iteration $k$. Moreover, this equation can be easily computed without solving a Lyapunov equation or approximating the solution as in the previous chapter, and is easier to compute the output variance by decoding with finite wordlength. In addition, we have derived the error (noise) matrix and error power gain for the measure of roundoff error.

When we round off coefficient $\boldsymbol{b}$, the output error variance is described as

$$\bar{V}_{\varepsilon}(k) = \frac{\sigma^2}{n^2} \sum_{l=1}^{n^2} \left( 2\, \frac{1-s_l^{2k}}{1-s_l^2} - 1 \right).$$

These analysis results, however, sacrifice the accuracy by about 1 dB.

In order to verify the method described, we computed the output error variance using the above equations and compared with the experimental output error variance. We tested the analysis method on several standard images that have 256×256 size or 512×512 size with 256 grey levels. Experimental results show that our modeling and analysis provide a good approximation for the standard images used.

## Chapter 5. Coefficient-quantization error analysis of fractal image coding

This chapter analyzes the coefficient-quantization error using the simplified state-space model used in Chapter 4. First, the model for coefficient-quantization error is given. Next, the analysis follows and goes into details by expanding the corresponding equations. The simulation results and the analysis results are compared for various types of standard images to verify the analysis method experimentally. Last, a summary is given for the obtained results.

In this chapter, we propose an analysis method of the coefficient-quantization error in fractal image coding using the state-space approach and the statistical method. We show that the statistical analysis method is appropriate and leads to a simple result, whereas the deterministic analysis method is not appropriate and leads to a complex results for the analysis of fractal image coding.

As an analysis result, we obtained the output error variance as follows:

$$\bar{V}_e(k) = \frac{\sigma^2}{n^2} \sum_{l=1}^{n^2} \left[ \sum_{j=0}^{k-1} s_l^{2j} \left\{ 1 + \left( \frac{1-s_l^{k-j-1}}{1-s_l} \right)^2 \right\} b_l^2 \right],$$

which consists of scaling coefficient $s_l$, iteration number $k$, and wordlength $L$. This equation is easier to compute than the output error variance in the experimentation. Moreover, we have derived the error (noise) matrix and error power gain for the measure of coefficient-quantization error.

In order to verify rhe method described in this chapter, we computed the output error variance using the above equation and compared with the experimental output error variance. We tested the analysis method on several standard images that have 256×256 size or 512×512 size with 256 grey levels. Experimental results show that our modeling and analysis are valid.

## 6. Conclusion and remarks

This chapter concludes this dissertation. Main analysis results of Chapters 3, 4, and 5 are summarized. Several remained problems are pointed out. Future research topics in the related fields are also introduced.

The main contributions of the present dissertation are summarized as follows:

1. The roundoff error and coefficient-quantization error for fractal image coding are mathematically analyzed using a state-space approach.
2. The main results obtained by the method proposed can be used to evaluate the efficiency of decoding process of fractal image coding.

3. The analysis method proposed can be applied to Jacquin-like coding schemes and can be applied to other schemes with appropriate modifications.
4. The method proposed can be extended to quantization error analysis of a deterministic iterated function system with appropriate modifications.

Future research topics are as follows.

1. Analysis of quantization error when the absolute value of the maximum scaling coefficients $s_{max}$ is larger than 1.
2. Analysis of quantization error of other coding schemes other than Jacquin's.

# 審 査 結 果 の 要 旨

フラクタル画像符号化は，単純な反復計算によって復元が可能であり，解像度に依存しない符号化が可能になるなどの優れた特徴を有している。しかし，反復計算に起因するさまざまな量子化誤差発生による複号画像の劣化が問題点の一つとされていた。

著者は反復関数系によるフラクタル画像符号化が，状態方程式によって系統的に表現できることに着目し，その量子化誤差の解析を行うと共に，語長と複合画像の精度の関係を明らかにした。本論文はこれらの成果をとりまとめたもので，全文6章よりなる。

第1章は緒言である。第2章では，反復関数系のフラクタル画像符号化の原理について述べると共に，符号化アルゴリズムの状態空間表現を与え，その基礎的性質を考察している。

第3章では，フラクタル画像符号化における丸め誤差解析を行っている。まず，状態方程式を用いて無限語長と有限語長のフラクタル画像符号化のアルゴリズムを表し，丸め誤差発生のモデルを与えている。このモデルを用いて，語長と複号画像の精度の関係を表す丸め誤差の評価式を導出し，評価値が実験結果と極めて良く一致することを確かめている。これは重要な成果である。

第4章では，前章で得られた成果に基づき，単純化された状態空間モデルを提案すると共に，丸め誤差評価が極めて効率良くできることを実証している。すなわち，著者は符号化における間引き率が，実際上極めて単純な値しか取らないことに着目することにより，状態空間モデルを簡単化できることを見出し，丸め誤差の評価式をスカラの閉じた形式で与えている。これにより，512 x 512の大きさの画像に対する丸め誤差が，直接的解析法に比べて約1／1000の極めて短い計算時間で評価可能になることを確かめている。これは有用な成果である。

第5章では，フラクタル画像符号化における係数量子化誤差の解析を行っている。フラクタル画像符号化では，それを表現する状態方程式の行列の次元が極めて大きくなることから，係数量子化誤差の解析には統計的方法が適していることを見出している。この統計的方法に基づき，係数量子化誤差の評価式を閉じた形で与えている。

第6章は結言である。

以上要するに本論文は，フラクタル画像符号化における量子化誤差の評価式を与え，語長と複号画像の精度の関係を定量的に明らかにすると共に，フラクタル画像符号化器設計のための有用な指針を与えたものであり，システム情報科学の発展に寄与するところが少なくない。

よって，本論文は博士（情報科学）の学位論文として合格と認める。